Retrieval-augmented Generation to Improve Math Question-Answering: Trade-offs Between Groundedness and Human Preference

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Abstract

For middle-school math students, interactive question-answering (QA) with tutors is an effective way to learn. The flexibility and emergent capabilities of generative large language models (LLMs) has led to a surge of interest in automating portions of the tutoring process—including interactive QA to support conceptual discussion of mathematical concepts. However, LLM responses to math questions can be incorrect or mismatched to the educational context—such as being misaligned with a school's curriculum. One potential solution is retrieval-augmented generation (RAG), which involves incorporating a vetted external knowledge source in the LLM prompt to increase response quality. In this paper, we designed prompts that retrieve and use content from a high-quality open-source math textbook to generate responses to real student questions. We evaluate the efficacy of this RAG system for middle-school algebra and geometry QA by administering a multi-condition survey, finding that humans prefer responses generated using RAG, but not when responses are *too* grounded in the textbook content. We argue that while RAG is able to improve response quality, designers of math QA systems must consider trade-offs between generating responses preferred by students and responses closely matched to specific educational resources.

1 Introduction

According to the National Assessment of Educational Progress (NAEP), nearly 40% of high school students lack a basic grasp of mathematical concepts [33], underscoring the need to enhance math education in K-12 environments. One of the most impactful methods to support students' math learning is through math question and answer (QA) sessions tutored by humans. Math QA can be approached with two main focuses: (1) enhancing students' *procedural* fluency with strategies such as step-by-step problem solving for specific math topics and (2) deepening students' *conceptual* understanding through scaffolding such as clarifying math concepts with concrete or worked examples, providing immediate feedback, and connecting math ideas to real-world scenarios [31, 40, 17]. While tutor-led math QA is effective [34], they face challenges such as efficiently allocating tutoring resources, ensuring wide accessibility due to high costs, and scaling up to support a myriad of learners with consistent quality [21, 11].

To address these challenges in math QA, educational researchers have sought AI to build expert systems and intelligent tutoring systems to enhance math learning with procedural practice [39, 4, 2]. However, limited educational research has focused on the potential of AI for improving students' conceptual understanding of math concepts. This study is a preliminary attempt to fill that gap by building the understanding needed to deploy conceptual math QA. We formed a research partnership

NeurIPS'23 Workshop on Generative AI for Education (GAIED).

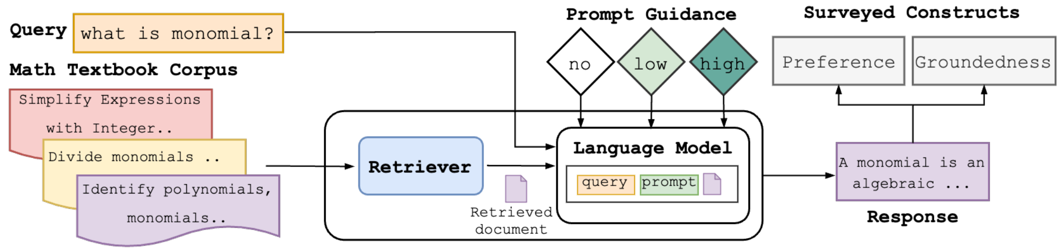


Figure 1: In this paper, we generated responses to math student queries with a retrieval-augmented generation system using one of three prompt guidance conditions. Survey respondents ranked responses by preference and assessed groundedness in the underlying math textbook used as a retrieval corpus.

with the developers of Rori, a WhatsApp-based chatbot math tutor primarily used by low-income middle-school students in Sierra Leone, Liberia, Ghana, and Rwanda.[1]. While Rori uses a chat interface, its pedagogical approach is based on intelligent tutoring systems (ITS) and it adopts a mastery-based learning approach that takes students through procedural lessons based on their abilities. Rori is currently designing for the inclusion of conceptual math QA using LLMs. There have been preliminary efforts to use LLMs in educational settings to scaffold student discussions, to provide feedback [20], to personalize learning experiences through automatic text analysis and generative socio-emotional support [46, 25], and to extend LLMs for many other educational tasks [43].

While the results from these education-related LLM explorations are encouraging, there are ethical considerations when using LLM outputs for math education [20, 35]. A primary concern is *hallucinations*, where LLMs generate answers that sound plausible and coherent but are factually incorrect [12]. Such misleading yet persuasive responses from LLMs could inadvertently instill incorrect conceptual understanding in students. Researchers from the AI community have investigated strategies to mitigate LLM hallucinations (see Ji et al.'s review [19]), with retrieval-augmented generation (RAG) standing out given its effectiveness and flexibility of implementation (e.g., model agnostic) [24, 52]. Conceptually, RAG in an educational context aims to bolster the correctness of LLM-based QA by drawing from external knowledge sources such as syllabi, workbooks, and handouts, such that the LLM's responses are, to various extents, anchored to established learning materials [36]. An interactive student chat backed by RAG offers the promise of both high correctness and faithfulness to materials in a vetted curriculum. Grounding tutoring materials in a student's particular educational context is an important requirement for system adoption [53, 16].

We implemented a RAG system for conceptual math QA (described in sec. 3). To evaluate our RAG system, we started with the problem of designing prompts that produce both the expected tutor-like behavior and responses grounded in the retrieved document. Can we use retrieval-augmented generation and prompt engineering to increase the groundedness of LLM responses? In study 1 (sec. 4), we observe qualitative trade-offs in response quality and the level of guidance provided in the LLM prompt, motivating quantitative study of human preferences. Do humans prefer more grounded responses? In study 2 (sec. 5), we survey preferences for LLM responses at three different levels of prompted guidance, finding that the most-preferred responses strike a balance between no guidance and high guidance. How does retrieval relevance affect response groundedness? In study 3 (sec. 6), we consider the impact of document relevance on observed preferences. Fig. 1 shows an overview of the RAG system and its use for addressing our research questions.

2 Related Work

Intelligent Tutoring Systems (ITSs) are educational technologies designed to provide one-on-one instructional guidance comparable to that of expert human tutors [37]. Structurally, ITSs implement a user interface over a knowledge base with a pedagogical model that determines how the ITS should respond to student inputs [41]. ITSs are traditionally based on iteratively serving procedural lesson content and providing hints in response to student mistakes [48]. ITSs have been shown to be effective

[1]`https://rori.ai`

Table 1: Representative student questions in the 51 Math Nation queries.

Can I get the steps for factoring quadratics	What is the domain and range? How do I find it?
How do I add line segments again??	How do you know if a number is a constant?
what is monomial	How do I multiply fractions???????

as tutors in specific domains such as mathematics and physics [49]. To extend an ITS that currently focuses on procedural fluency with features focused on conceptual understanding [45], we turn to the flexibility and expressive power of LLMs. LLMs have been proposed as useful for supporting a large number of education-related tasks [7, 20].

Despite the potential utility of LLMs for education, there are significant concerns around their correctness and ability to meet students at their appropriate level [20]. LLMs have been used in procedural tutoring and problem-solving systems, with careful prompt engineering used to improve reliability [47]. A more complex approach is using retrieval to augment the LLM prompt in order to improve response quality. For example, the SPOCK system for biology education retrieves relevant textbook snippets when generating hints or providing feedback [44]. Retrieval-augmented generation (RAG) involves retrieving texts from an external corpus relevant to the task and making them available to the LLM [24, 36]. RAG has been used to improve diverse task performance of LLMs [29], either by incorporating retrieved texts via cross-attention [18, 6, 24] or by inserting retrieved documents directly in the prompt [14].[2] We apply RAG in the education domain by using a math textbook as an external corpus and evaluating if RAG leads to responses that are preferred more often by humans and grounded in the textbook content.

3 A retrieval-augmented generation system for math question-answering

To support the development of reliable conceptual question-answering in a math chatbot, we implemented a retrieval-augmented generation system backed by a vetted corpora of math content, e.g. lesson plans, textbooks, and worked examples. RAG cannot provide a benefit during generation if the retrieved documents are not relevant, so we intentionally selected a corpus that will be relevant to many math-related student questions but not to all plausible questions.

OpenStax Prealgebra retrieval corpus We selected a Prealgebra textbook made available by OpenStax [28], segmented by sub-section. The textbook covers whole numbers, functions, and geometry, among other topics.

RAG implementation We adopted a commercially-realistic chatbot context as the underlying LLM, generating all responses with the OpenAI API using model gpt-3.5-turbo-0613 with default temperature settings. We built on our own implementation of RAG [23] that uses a variant of parent retrieval [8]. When a student asks a question, we identify a single relevant section of the textbook using cosine similarity against dense representations of the query and the textbook subsections. We created all representations using OpenAI's text-embedding-ada-002 model [13], an effective dense text embedding model [32]. We released our code and data on GitHub.[3] Additional details in App. A.

4 Study 1: Can we use retrieval-augmented generation and prompt engineering to increase the groundedness of LLM responses?

In using RAG, we hope that system responses will both answer the student's query and reflect the contents of the retrieved document. As the retrieved document cannot be perfectly relevant for all queries, achieving this *groundedness* may require producing inaccurate or otherwise less useful responses. Thus, there is an apparent trade-off between groundedness and the perceived usefulness of the system response. If this trade-off exists, we may want to influence the balance between

[2] A note on terminology: in Lewis et al.'s paper proposing "retrieval-augmented generation", they used the term to refer to an underlying LLM trained or fine-tuned with retrieved documents. The term has come to refer to any combination of LLMs and document retrieval: the method we use in this paper follows the common approach of using in-context learning rather than fine-tuning [22, 27]. A better term for these approaches may be "retrieval-enhanced machine learning" [55], which includes pre-LLM neural models using retrieval e.g. [9].

[3] `https://github.com/DigitalHarborFoundation/rag-for-math-qa`

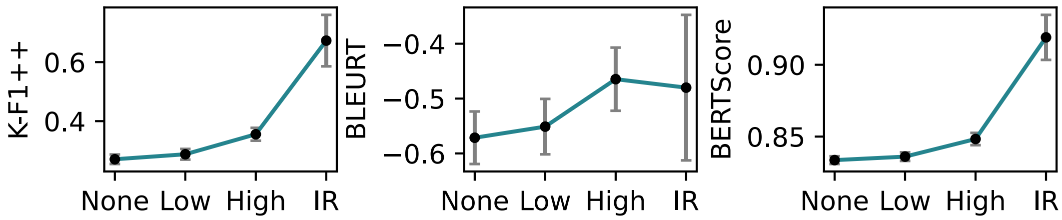


Figure 2: Groundedness for four levels of prompt guidance.

groundedness and usefulness by adjusting the system prompt. This first study tackles a basic question: *can* we influence this balance by engineering the prompt? We now introduce the prompt guidance conditions we used, the queries used for evaluation, and three evaluation metrics.

Guidance conditions Prompt engineering is important for LLM performance [30, 27, 47]. Each guidance condition was selected by iterative, qualitative exploration of prompts given 1-3 sample student questions. While these prompts are unlikely to be "optimal" [51], they produce reasonable outputs. The **No guidance** condition does not use RAG and contains a simple prompt that begins: "You are going to act as a mathematics tutor for a 13 year old student who is in grade 8 or 9 and lives in Ghana. You will be encouraging and factual. Prefer simple, short responses." Other prompts build on this basic instruction set—see App. B. The **Low guidance** prompt adds "Only if it is relevant, examples and language from the section below may be helpful to format your response:" followed by the retrieved document. The **High guidance** prompt instead says "Reference content from this textbook section in your response:". The **Information Retrieval** condition—used only in this first study to demonstrate the shortfalls of automated metrics for conversational responses—says "Repeat the student's question and then repeat in full the most relevant paragraph from my math textbook."

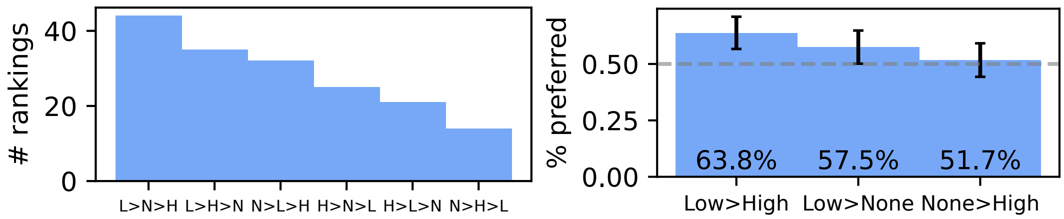
Student queries Math Nation is an online math platform with an interactive discussion board [5]. On this board, students seek help on math-related questions supported by their instructors, paid tutors, and peers. We annotated a random sample of 554 Math Nation posts made by students between October 2013 and October 2021 on boards for Pre-algebra, Algebra 1, and Geometry. We identified 51 factual and conceptual questions that have sufficient context to be answerable; the majority of excluded questions sought procedural help. Representative questions are shown in Table 1.

Evaluation metrics Given the relative novelty of our task, automatically measuring usefulness or correctness is not feasible. However, there is a large body of information retrieval (IR) literature on measuring groundedness of a generated text. We adopt three metrics used in prior work [1, 10, 12, 38]. K-F1++ is a token-level metric that completely ignores semantics, proposed by Chiesurin et al. as more appropriate for conversational QA than Knowledge F1 [10]. BERTScore is a token-level metric that uses RoBERTa-base embeddings to model semantics [56]. BLEURT is a passage-level metric that models semantics using BERT-base fine-tuned on human relevance judgments [42].

Results Fig. 2 shows that metric values on the 51 queries increase across guidance conditions. All confidence intervals are computed at the 95% significance level. These results confirm our basic intuition that groundedness is manipulable with prompt engineering. We do not know if response quality stays the same, increases, or even decreases as groundedness increases, but the results of the IR condition suggest that it *might* decrease: while the token-level metrics indicate that IR is the most grounded condition, its responses include no conversational adaptation to the student's question and so are lower quality in our context. In study 2, we will directly address the questions of response quality and groundedness by surveying humans.

5 Study 2: Do humans prefer more grounded responses?

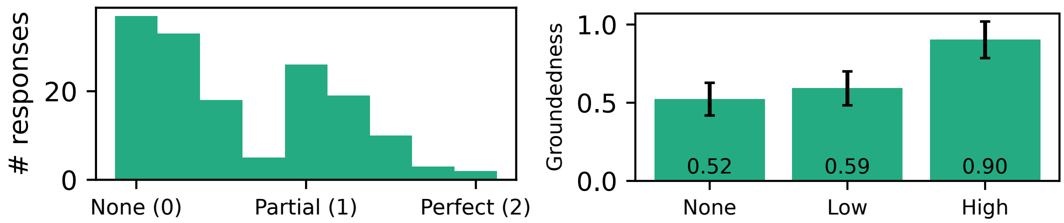
Methods To understand the impact of guidance on human preference for LLM responses, we surveyed 9 educators and designers of education technologies. We selected a comparative (within-subjects) design: with query and response order randomized, respondents ranked from best to worst the responses generated in the None, Low, and High guidance conditions for each query. To determine if the guidance conditions were perceived to be grounded in the retrieved document, we adapted a scale used in prior work as an ordinal None (0), Partial (1), Perfect (2) judgment [1]. Responses were spread across four Qualtrics surveys; all questions received 3-4 responses. The survey is in App. C.



(a) Distribution of all 144 preference rankings.

(b) Pair-wise preferences from all rankings.

Figure 3: Ranked preferences for LLM responses in three guidance conditions: no guidance (N), low guidance (L), and high guidance (H).



(a) Distribution of groundedness across responses in all three guidance conditions, averaged over annotators.

(b) Mean groundedness by guidance condition across all queries and annotators.

Figure 4: Groundedness of the generated responses on an ordinal None (0), Partial (1), Perfect (2) scale.

Results Fig. 3 shows respondent preferences for the three guidance conditions. Responses in the low guidance condition are preferred over responses in the no guidance *and* high guidance conditions. The high and no guidance conditions were statistically indistinguishable. At least two of the guidance conditions significantly differ in groundedness (n=153, one-way ANOVA F(2.0, 99.38)=6.65, p=0.001). We observed substantial inter-rater variation for groundedness ($n = 153$, Krippendorff's α=0.35). Fig. 4 shows that respondents do perceive high guidance responses to be more grounded in the retrieved document than low and no guidance responses. Surprisingly, low guidance responses are not perceived to be significantly more grounded than no guidance responses, suggesting that low guidance responses are preferred for reasons other than their groundedness, a question we will investigate further in study 3.[4]

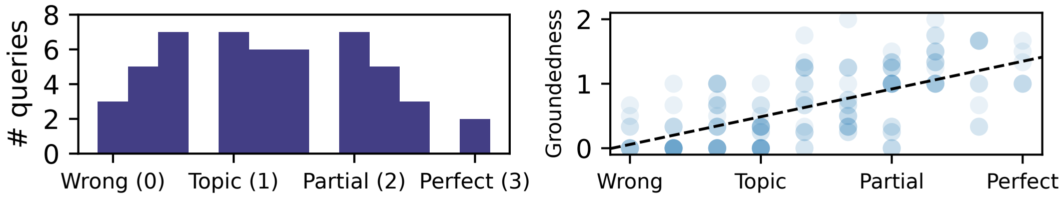
6 Study 3: How does retrieval relevance affect response groundedness?

Methods It may be that responses in the low guidance condition were preferred by survey respondents because the LLM includes content in the retrieved document if it is relevant and omits it if not. To test this hypothesis, three of the authors independently annotated each query and the associated retrieved document for relevance using a four-point ordinal scale used in prior work [15, 3]—see App. D.

Results Inter-rater reliability was generally low ($n = 51$, Fleiss' $\kappa = 0.13$, Krippendorff's $\alpha = 0.40$). For subsequent analysis, we computed the mean relevance of each document across annotators. 70.6% of queries are deemed at least topically relevant, while 33.3% are deemed partially relevant or better; see Fig. 5a for the full distribution. Across all guidance conditions, responses were more likely to be grounded if the retrieved document is relevant (Fig. 5b). However, we observed no significant relationship between relevance and preference (rank). For example, for queries where low guidance responses are preferred over high guidance responses, mean relevance is actually slightly *higher* (diff=0.19, t=-1.45, p=0.15).

Correlation between human annotations and automated metrics Given the results in study 2 suggesting that low guidance responses are not perceived to be more grounded than no guidance responses, we were further interested in possible correlations between perceived groundedness or

[4]Notably, there is no meaningful correlation between the rank of a low guidance response and its perceived faithfulness (Pearson's r=-0.08, p=0.29).



(a) Distribution of retrieved document relevance for all queries, averaged over annotators.

(b) Correlation between perceived groundedness and retrieved document relevance (r=0.56, p<0.001).

Figure 5: Human-annotated relevance of the retrieved document for all 51 queries.

Table 2: Correlation between human annotations and automated groundedness metrics. Pearson's r with p-values Bonferroni-corrected for 12 comparisons. Note: *p<0.05, **p<0.01, ***p<0.001.

Guidance	Faithfulness			Relevance		
	K-F1++	BLEURT	BERTScore	K-F1++	BLEURT	BERTScore
None	0.38	0.33	0.35	0.26	0.34	0.43*
Low	0.47**	0.32	0.61***	0.43*	0.34	0.50**
High	0.50**	0.21	0.39	0.37	0.26	0.50**
Pooled	0.52***	0.33***	0.51***	0.31**	0.30**	0.42***

relevance and the automated groundedness metrics. Table 2 shows modest positive correlations between automated groundedness metrics and human annotations. K-F1++ has the strongest correlation (r=0.52) with groundedness, although the correlation is weaker as guidance decreases.

7 Implications & Future Work

Across three studies, we investigated prompt engineering as a guidance mechanism alongside retrieval-augmented generation to encourage high-quality and grounded responses that are appropriate for students. Our most important finding is that **humans prefer responses to conceptual math questions when retrieval-augmented generation is used, but only if the prompt is not "too guiding".** While RAG is able to improve response quality, we argue that designers of math QA systems should consider trade-offs between generating responses preferred by humans and responses closely matched to specific educational resources. Math QA systems exist within a broader socio-technical educational context; the pedagogically optimal response may not be the one preferred by the student at that time. Chiesurin et al. distinguish between groundedness—when a response is found in the retrieved document—and *faithfulness*—when the response is both grounded and answers the query effectively [10]. Faithfulness is a desirable property for conceptual math QA systems, and we view designing for and evaluating faithfulness as an open problem. Our results show that prompt guidance with RAG is one potential design knob to navigate faithfulness. Future work might improve understanding of faithfulness by building taxonomies based on educational theories of effective tutoring, adapting existing procedural faithfulness metrics (e.g., [1, 12]), and explaining the role of retrieved document relevance (as in our surprising study 3 results finding that relevance was not a meaningful predictor of human preference).

This paper is a preliminary step toward understanding the relationship between groundedness and preference in conceptual math QA systems. Future work must extend beyond single-turn responses to include exploration of follow-up questions [50] and to design for the actual context of use. The most important limitation of this work is that we did not collect preferences directly from middle-school students, although we did use real student questions. Qualitative research of students' preferences should focus not only on correctness but also on factors such as conceptual granularity, curriculuar alignment, and cultural relevance. We were concerned about the ethics of presenting an untested math QA system to students but are now combining insights from these results with the implementation of guard-rails to deploy a safe in-classroom study. Beyond preferences, future math QA systems that use RAG will need to explore the relationship between students' response preferences and actual learning outcomes.

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A Implementation details

We opted to use GPT-3.5 rather than GPT-4 because it reflects a more realistic cost trade-off for the Rori ITS system we are researching. At the time of the study, GPT-3.5 had a context window of 4K tokens; we used up to 3K tokens for document retrieval. The median chapter and sub-section has 5,050 and 185 tokens respectively. We chose dense retrieval both for its popularity in RAG implementations and its dominance on a related retrieval task (not reported here) compared to a strong sparse-retrieval baseline: Pyserini's BM25 implementation [26, 54]).

B Prompts

Prompts used in the various guidance conditions. "{openstax_text}" is replaced with the retrieved text. The None, Low, and High guidance prompts are provided as system prompts, with the student question provided in a separate user prompt. The IR prompt is provided as a user prompt with "{query}" replaced by the student question.

B.1 No guidance (None) prompt

You are going to act as a mathematics tutor for a 13 year old student who is in grade 8 or 9 and lives in Ghana.

You will be encouraging and factual.

Prefer simple, short responses.

If the student says something inappropriate or off topic you will say you can only focus on mathematics and ask them if they have any math-related follow-up questions.

B.2 Low guidance (Low) prompt

You are going to act as a mathematics tutor for a 13 year old student who is in grade 8 or 9 and lives in Ghana.

You will be encouraging and factual.

Only if it is relevant, examples and language from the section below may be helpful to format your response:

===

Table 3: Number of unique queries annotated by each survey respondent.

Annotator	Query Count
A1	30
A2	30
A3	21
A4	21
A5	21
A6	15
A7	15
A8	15
A9	6

{openstax_text}

===

Prefer simple, short responses.

If the student says something inappropriate or off topic you will say you can only focus on mathematics and ask them if they have any math-related follow-up questions.

B.3 High guidance (High) prompt

You are going to act as a mathematics tutor for a 13 year old student who is in grade 8 or 9 and lives in Ghana.

You will be encouraging and factual.

Use examples and language from the section below to format your response:

===

{openstax_text}

===

Prefer simple, short responses.

If the student says something inappropriate or off topic you will say you can only focus on mathematics and ask them if they have any math-related follow-up questions.

B.4 Information Retrieval (IR) prompt

Given a middle-school math student's question, you will identify the most relevant section from a textbook.

Student question: {query}

Repeat the student's question and then repeat in full the most relevant paragraph from my math textbook. If none of them seem relevant, take a deep breath and output the most relevant. Don't say anything else.

Textbook paragraphs:

{openstax_text}

C Ranking & Groundedness Survey

Queries were split into four Qualtrics surveys; three surveys had 15 questions while the fourth had 6 questions. This section gives the exact survey text presented to respondents. 30 queries were annotated three times and the remaining 41 were annotated four times. Table 3 shows per-annotator counts.

C.1 Intro page

This survey will consist of 15 questions. Your progress will save after each question.

Who are you? (Annotator name) _________________

C.2 Query page

(Survey format note: this page is repeated once for each query in the survey.)

C.2.1 Ranking question

Rank these three responses from best to worst response. Consider if the response answers the question and is factually correct.

Student's question:

{query}

	1	2	3
{response1}	◯	◯	◯
{response2}	◯	◯	◯
{response3}	◯	◯	◯

C.2.2 Groundedness question

For each response, does the response or a paraphrase of the response appear anywhere in the following document?

Note: "First response" refers to the first response in the order they appear above, NOT the document you ranked as "1".

The document:

{openstax_text}

None: The response, even paraphrased, does not appear anywhere in the document.

Partial: Part of the response (or a paraphrase of the response) appears in the document.

Perfect: The response (or a paraphrase of the response) appears in the document.

	None	Partial	Perfect
First response	◯	◯	◯
Second response	◯	◯	◯
Third response	◯	◯	◯

C.2.3 Qualitative observation question

Notes/observations, if you want to flag something for later discussion with other annotators or if you spot a survey problem: _________________

D Relevance Survey

Three respondents (A1, A6, and A10) each independently annotated the 51 queries for relevance in separate tabs of a Google Sheet.

D.1 Annotator instructions

Each row contains a middle-school student's question (called the **query**) and an excerpt from a math textbook (called the **document**). Your task is to decide if the document is relevant to the query.

Your options are:

Wrong: The document has nothing to do with the query, and does not help in any way to answer it.

Topic: The document talks about the general area or topic of a query, might provide some background info, but ultimately does not answer it.

Partial: The document contains a partial answer, but you think there should be more to it.

Perfect: The document contains a full answer: easy to understand and it directly answers the question in full.

For readability, I bullet-pointed the paragraphs within each document. It's okay if only one paragraph within the document is relevant: if any paragraph within the document contains a full (or partial) answer, that is sufficient.

Each annotator has their own sheet within this workbook; annotate only within your own sheet, and don't look at others annotations.

D.2 Spreadsheet tab

The annotation sheet had the following columns: query, document, relevance